Physics: Interactions & Relationships

Physics is all about how properties of objects affect each other. Throughout the year, we will always seek a mathematical relationship between measurable quantities. This brief mathematical review will solidify your understanding of the backbone of how quantities relate: **direct** and **inverse** variation.

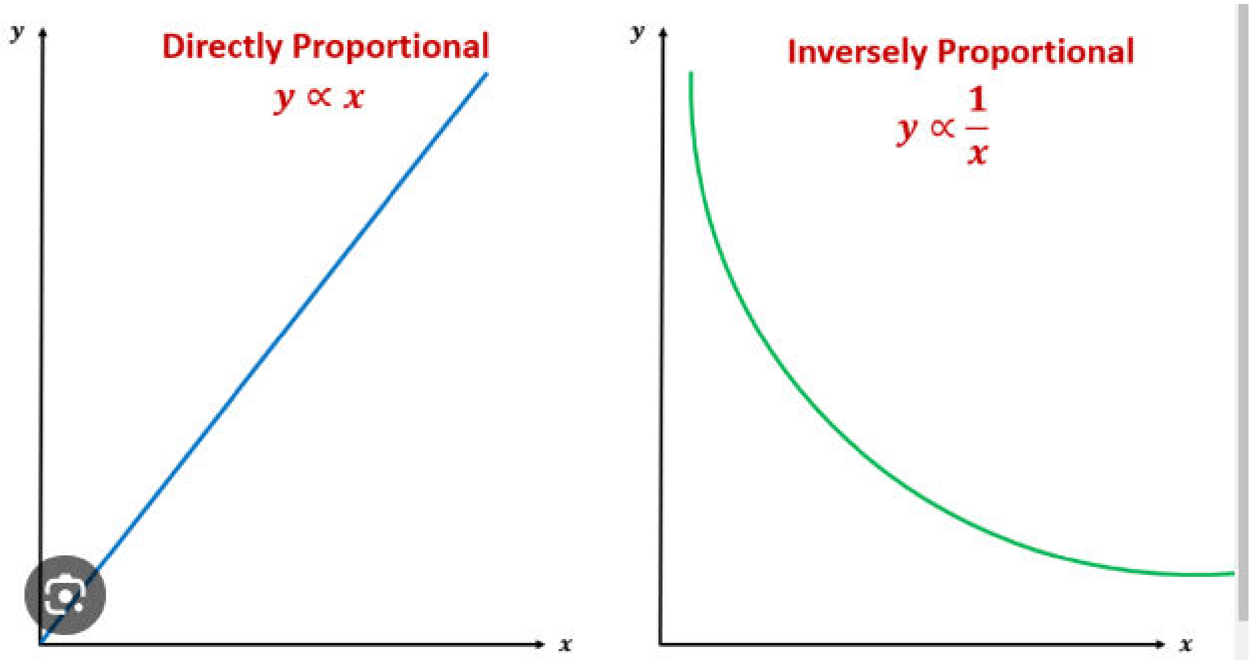


Figure 1: The graphs of direct and inverse variation.

Looking back: In chemistry…

You can find simple examples of direct and inverse variation all throughout chemistry. For example, consider the combined ideal gas law:

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ or $\frac{PV}{T} = k$ (this is a simpler formulation, stating that the product of pressure and volume divided by temperature never changes, always remaining equal to some number *k*)

When we talk about variation, we assume that only two quantities are changing, and all others are held **constant**. We ask the question, "What happens to one variable if we change the other, *assuming all other variables remain constant*?" Here, there are several instances of direct and inverse variation, depending on what quantities are changing:

P and V vary *inversely* (increasing one decreases the other by the same factor)
P and T vary *directly* (increasing one increases the other by the same factor)
V and T vary *directly* (increasing one increases the other by the same factor)

But in physics…

The math of variation can be more complicated as we might vary directly or inversely with and *expression* rather than with a single variable. Additionally, we might consider how a variable relates to either the *entire expression* or *just the other variable*. Consider the following real physics equation:

$$F = \frac{GMm}{r^2}$$

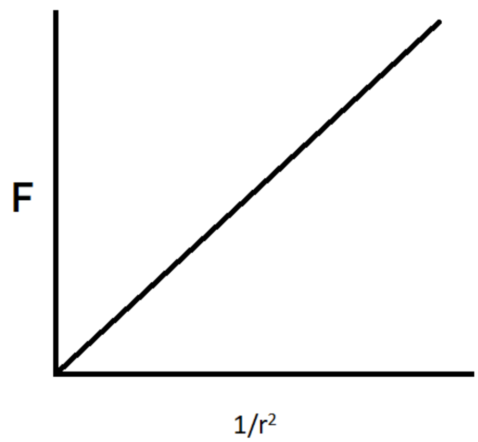
Some of the variation options here are simple. F relates directly to M or m; M relates inversely to m. However, when we consider how F relates to *r*, we may choose to describe that in several different ways, all of which are *equivalent descriptions* but *look different* and *graph differently!* The unfortunate reality of physics is that there are many ways to describe the same relationship and you have to be ready for any of them. Below are the options:

1) F relates directly to $\frac{1}{r^2}$
2) F relates inversely to r^2
3) F relates inversely to the square of *r* (see below for why this is different from option 2)

Let's explore each of those options: how they look, and what they mean.

Option 1: F relates directly to $\frac{1}{r^2}$

This description asks what happens to F when the *value of the expression* $1/r^2$ changes. They relate directly: if you double the *value of the entire quantity* $1/r^2$, then the value of F will also double. Graphing this relationship would show a diagonal line, but notice how the axes are labeled!

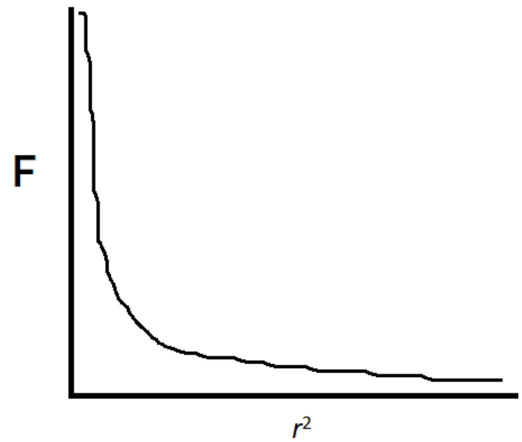


The values on the x-axis are not values of *r*; they are values of $1/r^2$. So if you have values of r, you would first calculate $1/r^2$ for each value of r, and those would be your x-values. For example, if r = 2, the x-value on this graph would be ¼. If r = 3, the x-value would be 1/9. Notice that as r increases, $1/r^2$ decreases, so the values of $1/r^2$ appear in the opposite order as compared to plain values of r.

This is called **linearizing** a graph. By choosing to graph $1/r^2$ on your x-axis rather than r, we have taken a relationship that would otherwise *not* be linear and made it into one that is linear. It is *much* easier to analyze linear relationships, so we like reframing equations into linear relationships whenever possible. See options 2 and 3 below for the nonlinear versions.

Option 2: F relates inversely to r^2

This description asks what happens to F when the value of the expression r^2 changes. They relate inversely: if you double the value of r^2, then you divide F by 2.



Once again, the values on the *x*-axis are not values of *r*. You have to compute the values of r^2 before graphing. For example, if your *r* values are 2, 3 and 4, you would graph 4, 9, and 16 as your x-values. The result is that if you double any x-axis value on this graph, then the y-axis value will divide by 2 (or more generally, if you multiply by any constant *k*, then you will divide the y-value by *k*).

What's the advantage here? Well, at least now the values of r^2 are in the same order as *r*. Sure, 4, 9, and 16 are spaced out much more than 2, 3, and 4, but they are still going to the right on the graph. We lose the nice straight line model, though, so there is a cost to describing the relationship this way.

Option 3: F relates inversely to the square of r
This description asks what happens to F when *r* changes. This might seem like the simplest way to analyze the relationship – and it certainly has its advantages – but in other ways, it is the most complicated and confusing mathematically. For starters, when *r* doubles, F does not divide by 2; it divides by the *square of 2*, or 4. More generally, if r multiplies by *k*, then F divides by k^2. Additionally, the graph of this relationship is eerily similar to the graph of the previous option (while not technically being the same) so I will not even bother confusing you by showing it. You will see it in the practice example.

In short, the problem with always relating one variable to another variable is that the relationship is not always simple. We like being able to minimize how much thinking we have to do, so keeping things as plain direct variation or plain inverse variation is often helpful. But to do that, we can't ask about F vs. r; we have to ask about F vs. r^2, or F vs. $1/r^2$, and modify what values we put on our axes.

Practice 1:

Graph the relationship $F = \frac{GMm}{r^2}$ in three different ways using the constant values G = 2, M = 4, and m = 8:

Graph F vs. r

Graph F vs. r^2

Graph F vs. $1/r^2$

For each graph, make a small table of values using whole number values of r between 1 and 5. Consider carefully how to label your x-axis in each case so that the graph is always visible (e.g. if all your x-values are under 1, don't label your axis from 1 to 5…). **USE GRAPH PAPER. You are plotting precise points, not a general sketch.** Helpful tip: Convert fractions to decimals so you can graph evenly.

Practice 2:

Consider the variables *d* and *t* in the equation $d = \frac{1}{2}at^2$

1) If you triple the value of t^2, what is the effect on *d*?
2) If you triple the value of *t*, what is the effect on *d*?
3) If you graph this with *d* on the y-axis and *t* on the *x*-axis, what shape will it have?
4) If you graph this with *d* on the *y*-axis and t^2 on the *x*-axis, what shape will it have?
5) One of the previous two questions' answer is "a diagonal line." For that one, what is the slope of that line, in terms of *a*?

Practice 3:

Consider the variables *v* and E in the equation $v = \sqrt{\frac{2E}{m}}$

1) If you quadruple the value of E, what happens to *v*?
2) If you quadruple the value $\sqrt{E}$, what happens to *v*?
3) Fill in the blank: If you graph this with *v* on the *y*-axis and ____________ on the *x*-axis, it will form a diagonal line with a slope of _________ (in terms of *m*).

Answers to Practice 2 and Practice 3 are on the next page. Make sure you understand what you are doing! **If you are unable to make sense of the mathematical concepts and you just copied the answers, I would suggest considering a different physics class, because this year will be <u>very</u> hard for you. You are also welcome to email me at segallm@mjbha.org if you have questions. I would *love* to help you – but please, *please, <u>please</u>* do not come to class on day 1 without understanding this summer assignment!**

Answers to Practice 2:

1) *d* triples

2) *d* multiples by 9

3) A parabola

4) A diagonal line

5) $\frac{1}{2}a$ [match this up to the structure $y = mx$. x is t^2; the slope m is everything attached to the x]

Answers to Practice 3:

1) It gets multiplied by the square root of 4, or 2.
2) It gets multiplied by 4.
3) If you graph this with *v* on the *y*-axis and $\underline{\sqrt{E}\quad\quad}$ on the *x*-axis, it will form a diagonal line with a slope of $\sqrt{\frac{2}{m}}$ [match this up to the structure $y = mx$. x is $\sqrt{E}$; the slope m is everything attached to the x]